

# Physical-Layer Network Coding: An Efficient Technique for Wireless Communications

Pingping Chen, Zhaopeng Xie, Yi Fang, Zhifeng Chen, Shahid Mumtaz, and Joel J. P. C. Rodrigues

**Abstract**—As a subfield of network coding, physical-layer network coding (PNC) can effectively enhance the throughput of wireless networks by mapping superimposed signals at receiver to other forms of user messages. Over the past twenty years, PNC has received significant research attention and has been widely studied in various communication scenarios, e.g., two-way relay communications (TWRC), nonorthogonal multiple access (NOMA) in 5G networks, random access networks, etc. Later on, channel-coded PNC and related communication techniques were investigated to ensure network reliability, such as the design of channel code, low-complexity decoding, and cross-layer design. In this article, we briefly review the variants of channel-coded PNC-aided wireless communications with the aim of inspiring future research activities in this area. We also put forth open research problems along with a few selected research directions under PNC-aided frameworks.

## I. INTRODUCTION

In the fifth generation (5G) wireless communication networks, non-orthogonal multiple access (NOMA) has been attracting significant research efforts for the design of radio access techniques [1]. However, NOMA suffers from degraded spectral efficiency due to signal interferences in simultaneous user transmissions. To overcome this problem, many coding and signal processing techniques were proposed to mitigate and utilize the multiuser interference. The concept of physical-layer network coding (PNC), inspired by traditional network coding, was proposed and demonstrated its advantages over the conventional communications from both information-theoretic and practical perspectives [2]. In particular, PNC can significantly improve the network throughput by exploiting the characteristic of user interferences. Moreover, those design schemes based on PNC are becoming competitive solutions for NOMA in 5G networks [3]. In PNC-aided networks, the receiver is dedicated to decoding linearly weighted combinations of user messages, referred as network-coded (NC) message, from the received signals. A simple PNC operated network is two-way relay channel (TWRC), in which two user nodes desire to communication with each other via a relay. A PNC-aided TWRC has two phases. The first phase is multiple access phase and the second one is broadcast phase. As illustrated in Fig. 1, in the first phase, user 1 sends message  $S_1$  and user 2 sends message  $S_2$  to the relay simultaneously. Given the

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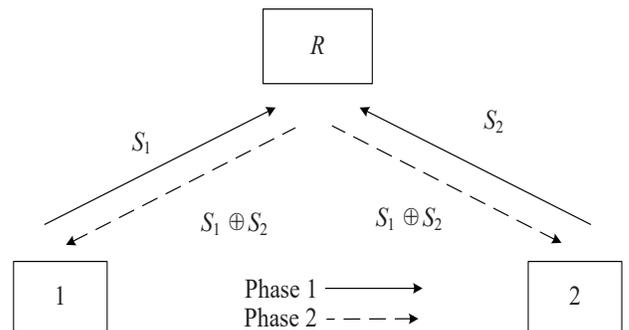


Fig. 1. Physical-layer network coding.

superimposed message from two users, the relay attempts to decode a linear combination of  $S_1$  and  $S_2$ ,  $S_1 \oplus S_2$ . Then, in the second phase, the relay broadcasts  $S_1 \oplus S_2$  to the two users. By doing so, PNC was shown to outperform the conventional transmission scheme in practical scenarios in terms of sum-rate and decoding performance [4], [5].

As an emerging technique, PNC brings up many issues in a variety of wireless communications scenarios. Interesting issues include how to characterize the decoding behavior of PNC in power-imbalance TWRC [6], and how to attain full-rate and full-diversity in PNC-aided multiple-input multiple-output (MIMO) scenarios [7]. An information-theoretic issue is how to design powerful channel coding to approach the PNC cut-set bound on the information capacity. The lattice-coded PNC has been shown to achieve a small gap less than 1/2 from the cut-set bound in power-imbalanced Gaussian channels [8]. However, high encoding/decoding complexities hinder the practical implementation of lattice-coded PNC. Driven by this issue, recent interest and effort has been devoted to binary/nonbinary channel-coded PNC.

To our knowledge, the research of nonbinary channel-coded modulation PNC (CM-PNC) falls into three general methods, i.e., network coding-based channel decoding (NC-CD), multiuser complete decoding (MUD-NC), and channel decoding for nonbinary physical-layer network coding (CD-NC) [9], [10]. Accordingly, the binary channel-coded, i.e., bit-interleaved coded modulation (BICM), counterparts of NC-CD, MUD-NC, and CD-NC are XOR-based channel decoding (XOR-CD), MUD-XOR, and CD-XOR, respectively [11]–[13], as summarized in Fig. 2. Note that XOR is a form of NC which is usually used to obtain bitwise XOR messages. Moreover, an interesting phenomenon is that MUD-NC outperforms NC-CD and CD-NC in the low SNR regime in terms of the achievable

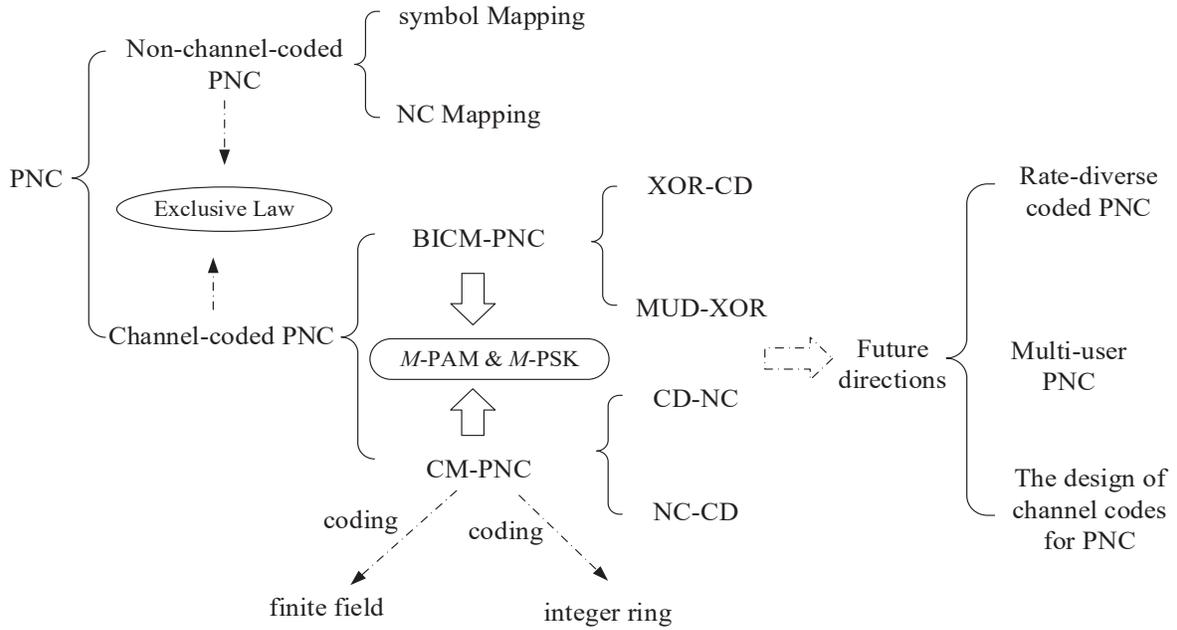


Fig. 2. Organization of the paper.

rate, and vice versa in the high SNR regime. This comparison also holds for their binary channel-coded counterparts.

NC-CD/XOR-CD first maps superimposed channel-coded symbols from two users to NC symbols before explicit channel decoding. Thus, the relay can directly decode channel-coded NC message without knowing individual user messages. By contrast, MUD-NC/MUD-XOR first decodes two user messages given the superimposed signals by MUD, and then performs NC mapping on two decoded messages [4]. Unlike the previous schemes, CD-NC/CD-XOR first maps superimposed signals to a transmit symbol pair and then performs channel decoding. Afterwards, we can obtain NC message based on the decoded transmit pairs.

For XOR-CD, [14] developed XOR-CD with non-uniform PAM constellation to facilitate the use of BICM for PNC. While for PSK modulation, [15] investigated the design the optimal bit mapping of PSK symbols for BICM-PNC. Moreover, [12] showed that the achievable rate of XOR-CD is smaller than that of its NC-CD counterparts even under Gray-mapped high-order modulations, and proposed an iterative XOR-CD to narrow this rate gap. For NC-CD, [9], [12] considered low-density-parity-check (LDPC) coding aided PNC over the finite fields  $\mathbb{F}_3$ ,  $\mathbb{F}_{2^2}$  and  $\mathbb{F}_5$ . Nevertheless, different superimposed signals before decoding are mapped to the same NC symbol degrades the performance of channel decoding. For this reason, [10] proposed CD-NC to overcome this issue. CD-NC employs nonbinary LDPC codes over the integer ring  $\mathbb{Z}_M$  rather than the finite field, and a novel generalized decoding algorithm was developed accordingly.

In this article, we give a general overview of channel-coded PNC in TWRC. We first present the relevant system models and the NC mapping for PNC. Afterwards, we describe the main encoding and decoding schemes for channel-coded PNC. With simulation results, we illustrate the decoding performance of different channel-coded PNC schemes over AWGN and

Rayleigh fading channels. To the best of our knowledge, this is the first tutorial touching upon *channel-coded PNC* for the relay networks. Interested readers are referred to the references for more comprehensive treatments of this promising topic.

## II. SYSTEM MODEL

In the multiple-access phase of a TWRC, two users encode their source messages using the same channel code to yield two codewords, respectively. Then two signal vectors are created by  $M$ -ary modulation on the two codewords, respectively. The two modulated vectors are then simultaneously transmitted to relay R. It is pointed out that the goal at the relay is to decode the NC message from the superimposed signals. The NC message is then encoded for broadcast to the two users. The key point in PNC is how to decode the NC message from the superimposed signals in the first phase.

In such a channel-coded PNC, the desired NC vector is a valid codeword of the encoder and can be decoded with the same decoder. We consider  $M$ -PSK and  $M$ -PAM modulations to study at user nodes. The cardinality of both constellation sets of the two users are the same. As a result, we have the total  $M^2$  joint symbol pairs for transmission by treating two user symbols as a transmission pair. Accordingly, the receiver can form a superimposed constellation set, in which each element is a linearly superposition of two modulated symbols weighted by their user-to-relay channel coefficients, respectively.

In general, in a TWRC network over fading channels, each transmission pair generates a superimposed signal weighted by fading coefficients. Thus, the superimposed set has  $M^2$  elements and each element is distinct. On the other hand, in a TWRC over AWGN channels where two user-to-relay channel coefficients are the same, the number of elements in the superimposed set is less than  $M^2$ . This is because that multiple transmit pairs may lead to the same superimposed signal. Recall that the goal of PNC is to compute the NC

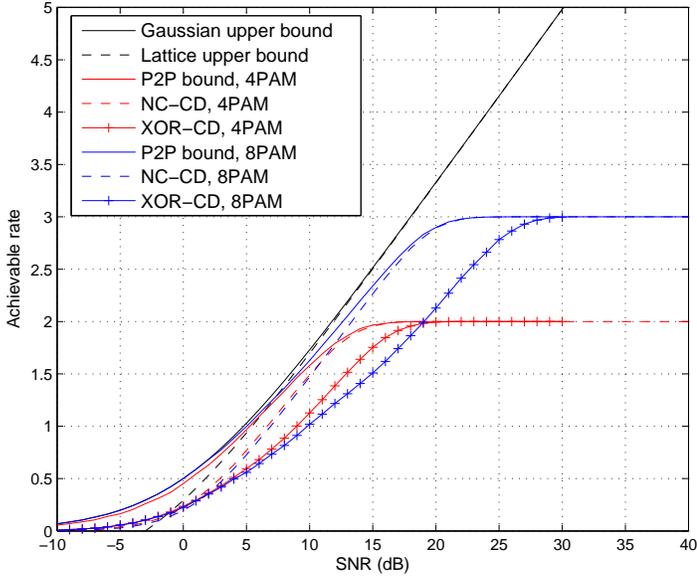


Fig. 3. PNC-rate comparison of XOR-CD and NC-CD over AWGN channels.

message out of the superimposed signal at the relay. It turns out that we should give the NC mapping such that a unique NC symbol can be derived from the superimposed signal without ambiguity.

#### A. Non-channel-coded PNC

In non-channel-coded PNC, we treat an NC mapping as feasible as long as the mapping satisfies the *exclusive law* [14], [15]. Each of the two users can be ensured to retrieve the message of the other user given its self-information and the received NC message in the broadcast phase. For example, for  $M$ -PAM based PNC, a feasible NC mapper is the modulo- $M$  arithmetic of two transmit symbols over integer ring  $\mathbb{Z}_M$ . With respect to  $M$ -PSK based PNC, a feasible mapper is an addition of two transmit symbols over the finite field  $\mathbb{F}_{2^r}$ ,  $2^r = M$ . Note that both the demappers can be performed with different  $M$ -ary decoding coefficients which belong to a set of non-zero divisors. In this way, we can obtain a unique NC symbol derived from the superimposed symbol which is generated even by multiple transmission pairs. The choice of decoding coefficients has been intensively studied in [6].

It is pointed out that for some channel-coded PNC, it is not strictly required for the NC mapping that all mapped NC symbols cannot have mapping ambiguity. This is because that channel decoding can recover the corrupted NC symbol of ambiguity in some extent. Even so, the ambiguity in the channel decoding still undergoes a rate loss theoretically. For this reason, the channel-coded PNC abides by the same design rule of the NC mapping as non-channel-coded PNC. We next delve into the details of BICM-PNC and CM-PNC.

#### B. BICM-PNC

In practice, the decoding complexity of CM-PNC using nonbinary codes is prohibitively increased as the size of coding

symbol increases. Thus, to achieve high spectral efficiency and low complexity, we consider BICM-PNC with high-order modulation as an alternative to lattice-coded/nonbinary-coded CM-PNC. For BICM-PNC, we have to select the proper mapping at the user nodes and the NC mapping at the relay. In some channels (e.g., AWGN channels), one received symbol in the superimposed constellation may be associated with more than one transmit symbol pair, which may lead to different NC symbols, i.e., mapping ambiguity. From this view of point, we should carefully determine bit mappings of high modulations to avoid ambiguity.

Consider XOR-CD and MUD-XOR for BICM-PNC. For  $M$ -PAM based XOR-CD, a non-uniform PAM with modified spacing between PAM constellation points was proposed as legitimate bit mappings [14]. For  $M$ -PSK modulations, [15] carefully analyzed the symbol mapping at user nodes, and proposed a semi-Gray mapping to achieve the best performance in the high SNR regime.

Moreover, BICM capacity and CM capacity for Gray-mapped point-to-point communication system are almost the same. By contrast, for both  $M$ -PSK and  $M$ -PAM PNC systems, we found that XOR-CD and MUD-XOR suffer an appreciable capacity loss from their CM counterparts, i.e., NC-CD, and MUD-NC, respectively, even with Gray-mapping modulations [10], [12]. To narrow this loss, we proposed an iterative decoder for BICM-PNC where the messages between bits-to-symbol PNC mapper and channel decoder are further iteratively exchanged and updated [12].

#### C. CM-PNC

Although XOR-CD is a simple channel-coded PNC scheme, it has a significant rate gap from nonbinary-coded CM-PNC with high modulations, as shown in Fig. 3. We consider  $M$ -PSK based and  $M$ -PAM based CM-PNC with the corresponding  $M$ -ary channel coding schemes, respectively. Similarly, the NC mapping in CM-PNC needs to satisfy that the NC vector of two user codewords is still a valid codeword. CM-PNC is also applicable for a TWRC, in which two users adopt  $M^2$ -QAM modulations, since any QAM signal can be treated as two  $M$ -PAM orthogonal real and imaginary parts. We can directly apply the mapping and coding methods in each part.

In CM-PNC, we emphasize that the operation of channel coding should be consistent with the NC mapper. Recall that the NC mapper for  $M$ -PSK based PNC is an addition of two user symbols over the finite field  $\mathbb{F}_{2^r}$ , and for  $M$ -PAM based PNC is a mod  $M$  operation of two user symbols over the integer ring  $\mathbb{Z}_M$  [11], [15]. Based on this observation, for  $M$ -PSK based CM-PNC, we adopt  $M$ -ary channel coding over  $\mathbb{F}_{2^r}$ . On the other hand, for  $M$ -PAM based CM-PNC, we adopt  $M$ -ary channel coding over  $\mathbb{Z}_M$  instead of  $\mathbb{F}_{2^r}$ . In this way, we can keep the consistency between the decoding operation and the NC mapping operation, which involve multiplications and additions. Also, decoding coefficients used in the NC mapping are selected from non-zero divisors in  $\mathbb{F}_{2^r}$  and  $\mathbb{Z}_M$ , for  $M$ -PSK and  $M$ -PAM based CM-PNC, respectively. Such consistency is essential for the NC decoding to extract the desired messages at two users. We summarize the NC mapping rules for channel-coded PNC systems as follows:

- 1) NC mapping must satisfy the *exclusive law*.
- 2) NC mapping cannot incur ambiguity, i.e., each superimposed symbol is mapped into a NC symbol uniquely.
- 3) The entire mapped NC codeword is a valid codeword of the channel code adopted at the user nodes.

### III. DECODERS OF CHANNEL-CODED PNC

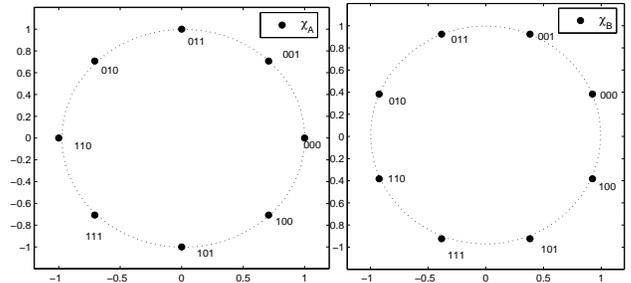
#### A. XOR-CD for BICM-PNC

Recall that we have two decoders, i.e., MUD-XOR and XOR-CD for BICM-PNC. In specific, XOR-CD first performs symbol-by-symbol bitwise XOR to obtain NC symbol from the received signals since binary channel code is used at two user nodes. Then, the symbol-to-bits demapper computes the soft information about the XOR bits from the NC symbol for the channel decoder. Note that if multiple possible transmit symbol pairs results in the same received signal, it may generate different bitwise NC symbols. In such a case, an ambiguity occurs and useful information about the NC symbol is lost, in that we cannot distinguish which of the NC symbols is correct. As a result, we have to carefully design the transmit constellations of two users to avoid ambiguity [10], [12].

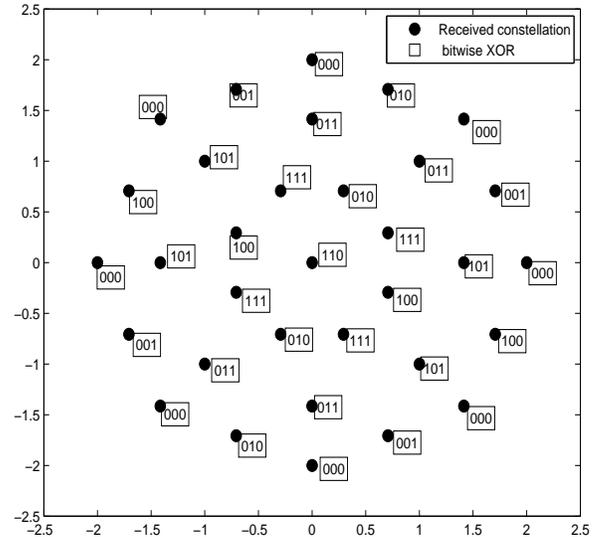
For example, Fig. 4(a) shows two different Gray-mapped 8PSK constellation sets  $\chi_A$  and  $\chi_B$  for two users respectively, where  $\chi_B$  has a rotation of  $\pi/8$  from  $\chi_A$ . In this case, we have 64 elements in the superimposed set at the relay. Each element is generated by a unique transmission pair and a unique NC symbol can be determined from the element. It implies that the constellation sets are applicable for XOR-CD. Moreover, consider an XOR-CD where the two users employ the same set  $\chi_A$  or  $\chi_B$  in Fig. 4. The superimposed set has 33 elements because some transmission pairs yield the same superimposed element. Fortunately, from Fig. 4 (b), we see that the ambiguous transmission pairs still yield the same bitwise NC message. Therefore, we conclude that this strategy can also be used for XOR-CD.

#### B. Iterative XOR-CD for BICM-PNC

For BICM, different bit-to-symbol mappings of the constellation lead to different channel capacities. It can approach the CM capacity if the bit positions in one symbol are considered as independent. The BICM capacity with Gray mapping is close to the CM capacity for point-to-point systems. It is not necessary to introduce iterations between channel decoder and symbol-to-bits demapper due to little gain achieved by iterations. Nevertheless, for XOR-CD with Gray mapping at two users, the demapped XOR bits within NC symbols from the superimposed constellation at the relay are not completely Gray-mapped or independent. This can be exemplified from Fig. 4 (b) that the XOR bits within the received constellation for 8PSK XOR-CD are not Gray-mapped. Based on such an alarming observation, we consider outer iterations between the binary channel decoder and the NC symbol demapper, which outperforms its non-iterative counterpart. The performance gain is verified by extrinsic information transfer chart analysis and distance spectrum analysis [12]. Also, we indicate that the information capacity of iterative XOR-CD can approach that of the NC-CD counterpart.



(a) Two different constellation sets for two users respectively, where the dots denote the modulated signals. Two constellations have a angle rotation of  $\pi/8$  from each other.



(b) Received constellation from the same user constellations

Fig. 4. Gray-mapped 8PSK constellation sets and the bitwise NC symbols in channel-coded PNC.

#### C. NC-CD for CM-PNC

Similar to XOR-CD, NC-CD deals with a symbol-by-symbol NC mapping and then performs channel decoding. As mentioned before, two users employ the same nonbinary channel coding for  $M$ -PAM and  $M$ -PSK based CM-PNC. In such scenarios, NC-CD tries to decode multiple NC vectors with different  $M$ -ary coefficients from the received signal, which enhances the probability for successfully decoding.

As an example, we elaborate the NC-CD for  $M$ -PAM based CM-PNC. Given decoding coefficients, we first compute the initial  $M$  soft messages for each NC symbol. The computed messages are then fed into a conventional sum-product algorithm (C-SPA) because the entire NC vector is a codeword of the nonbinary coding. Note that there are total  $M^2$  decoding coefficient pairs in NC mapping. It implies that we can implement C-SPA for one NC vector with a pair of coefficients at most  $M^2$  times, leading to the increased decoding complexity, as only one of the successfully decoded NC vectors suffices for our systems. To address this issue, given the fixed channel coefficients, we can decide the best decoding coefficient pair rather than all the pairs for decoding by maximizing the effective minimum distance between NC

mapped symbols [6]. Nevertheless, the selection of the decoding coefficients also increases computational complexity and cannot work well when the channel coefficients vary over the codeword transmission.

#### D. CD-NC for CM-PNC

Although the achievable rate of NC-CD is larger than that of XOR-CD, it is still far away from the cut-set bound of TWRC theoretically. For example, consider a TWRC with  $M$ -PAM modulations over Rayleigh fading channels, we remark that there exists a distinct rate loss of more than 5 dB between NC-CD and upper bound when  $M \geq 4$ , for both low and high SNR regimes [10]. To mitigate the loss, we propose CD-NC with a generalized SPA (G-SPA) algorithm to decode the NC vector. In specific, we first virtually construct an encoder where transmit pair symbols are encoded based on the encoding rule at the users and the superposition rule at the relay. Subsequently, we derive check node update and variable node update in G-SPA according to the constructed encoder. Unlike NC-CD, CD-NC first implements channel decoding for the transmission pair symbols and then performs the symbol-by-symbol NC mapping. In G-SPA, we first compute  $M^2$  probabilities of  $M^2$  transmission pairs from the two users, which are then iteratively updated with the derived update rules. Afterward, we can decide the transmission pair of the highest probability for each NC symbol, which is finally used for NC mapping to produce the NC symbol.

Revisiting NC-CD, we present the main differences between NC-CD and CD-NC as follows:

- 1) NC-CD first calculates the initial soft messages of the mapped NC symbols, and then performs SPA to directly decode the NC vector. In contrast, CD-NC starts with the decoding for all transmission pairs for each NC symbol, and then performs NC mapping after decoding.
- 2) NC-CD is implemented at most  $M^2$  times, since multiple NC vectors with  $M^2$  decoding coefficients are possibly decoded, while CD-NC is implemented one time. Nevertheless, considering a single decoding process, C-SPA for NC-CD has lower decoding complexity than G-SPA for CD-NC, because the numbers of updated messages per symbol in C-SPA and G-SPA are  $M$  and  $M^2$ , respectively.

For SPA decoding, the operation in the  $d$ -degree check node dominates the decoding complexity. As mentioned above, G-SPA iteratively updates  $M^2$  messages for each NC symbol, with the computational complexity being  $\mathcal{O}(d\theta^2)$ ,  $\theta = M^2$ . This complexity becomes larger as  $M$  increases. In point-to-point communications, we prefer to adopt fast-Fourier-transform (FFT) based algorithm of lower decoding complexity. Similarly, we proposed two-dimensional (2D)-FFT based BP as a simplified alternative of G-SPA [10]. The computational complexity of 2D-FFT-BP is reduced to  $\mathcal{O}(d\theta \log_2(\theta))$  from  $\mathcal{O}(d\theta^2)$  in G-SPA. Moreover, we also apply the extended min-sum (EMS) decoding which only involves additions for CD-NC to avoid the multiplications and divisions for message normalization in 2D-FFT. By doing so, the decoding complexity can be further reduced to  $\mathcal{O}(dn_m \log_2(n_m))$  and  $n_m$  is a configure parameter in EMS [10].

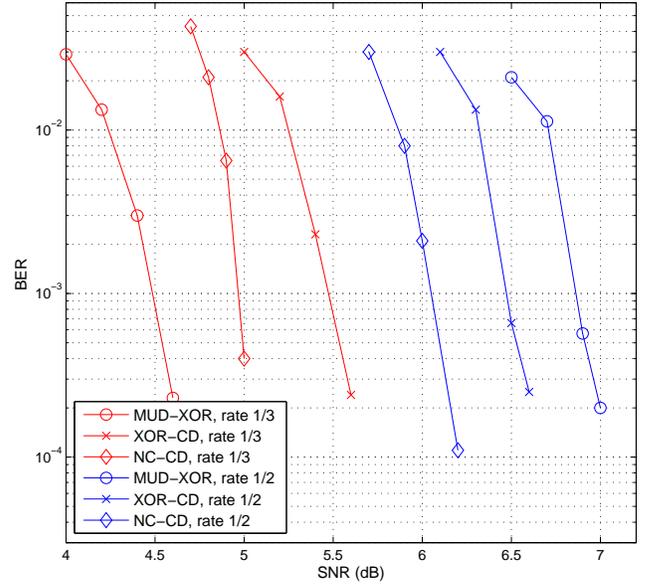


Fig. 5. BER performance of XOR-CD, NC-CD, and MUD-XOR with 8PSK over AWGN channels at rates of 1/3 and 1/2.

#### E. MUD-NC/MUD-XOR

For MUD-NC and MUD-XOR, the relay first demaps two user codewords from the received signals individually. Thus, if multiple transmission pairs give to the same superimposed signal, an ambiguity occurs and the achieved rate is reduced in that we cannot distinguish two user symbols uniquely. Therefore, a constellation pair for the two users should be designed such that each superimposed signal corresponds to a unique transmission pair. Intuitively, two constellation sets cannot be the same. In fact, for  $M$ -PSK case, a constellation pair is required to possess an appropriate angle rotation between them, while for  $M$ -PAM case, the constellation pair has the different spaces between the constellation points.

For example, consider two different Gray-mapped 8PSK constellations for two users, as shown in Fig. 4(a). Then the resulting superimposed set has 64 elements, and each corresponds to a unique transmission pair, which is suitable for MUD-XOR/MUD-NC. However, if both users adopt the same set, some elements in the superimposed set may correspond to more than one transmission pairs. In other words, the same constellation set at the users introduces ambiguity without yielding uniquely decodable constellation pair. It implies that we cannot apply this strategy for MUD-XOR/MUD-NC.

In MUD-XOR, we first compute the soft information of two transmit symbols from the two users. Then, the symbol-to-bits demapper computes the soft information of the two coded bits for their respective binary decoders. After channel decoding, the relay achieves a bitwise XOR message on the two decoded codewords. Moreover, we can further introduce iterative decoding for MUD-XOR where extrinsic information from one user decoder can be utilized by the other one user decoder. As a result, the decoding performance of MUD-XOR can be improved.

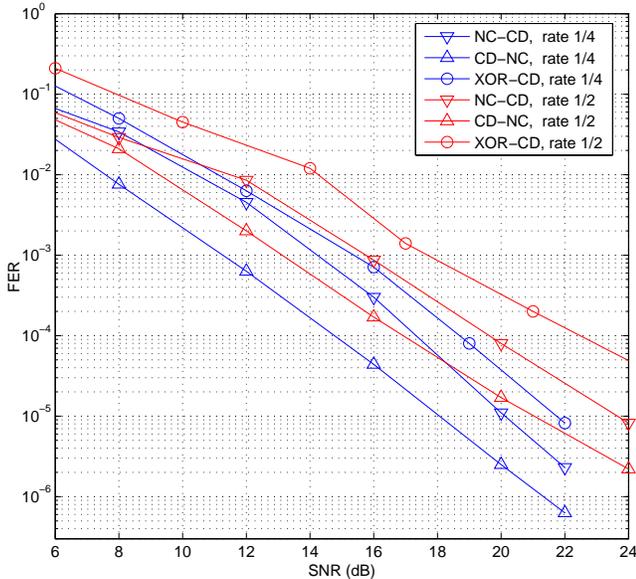


Fig. 6. FER performance of XOR-CD, NC-CD, and CD-NC with 4PAM over block fading channels at rates of 1/4 and 1/2.

#### IV. PERFORMANCE RESULTS

To evaluate the performance of channel-coded PNC, we present numerical results of the BICM-PNC and CM-PNC systems under LDPC channel coding using VC++ software. The information length of the codeword is 1032 for all simulations. The maximum number of decoding iterations is 150. For a fair comparison, in iterative XOR-CD and MUD-XOR, the numbers of out iterations and channel decoding iterations are assumed as 6 and 25, respectively.

In particular, we adopt the binary LDPC codes which were optimized for point-to-point communications, for BICM-PNC, i.e., XOR-CD and MUD-XOR. For CM-PNC, i.e., NC-CD and CD-NC, we adopt regular nonbinary LDPC codes. We do not show the performance of CD-XOR since it is a special binary case of CD-NC.

First, we consider PSK-based channel-coded PNC over AWGN channels where the relay aims for the bitwise XOR of two codewords from the two users. Fig. 5 plots the BERs of NC-CD, Gray-mapped MUD-XOR and XOR-CD with 8PSK modulation at code rates of 1/2 and 1/3, respectively. We assume a nonbinary LDPC regular code over  $\mathbb{F}_{2^3}$  for NC-CD where coding coefficients are randomly selected from  $\mathbb{F}_{2^3}$ . We can observe that NC-CD performs the best among the three schemes, and XOR-CD achieves a gain of 0.8 dB over MUD-XOR at a rate of 1/2. Conversely, MUD-XOR is superior to both XOR-CD and NC-CD at lower rate of 1/3. The results corroborate theoretical rate analysis that MUD-XOR tends to be better than XOR-CD in the low rate region, while XOR-CD becomes better in the high rate region. Moreover, NC-CD performs better than XOR-CD for both the code rates due to bits-to-symbol mapping in XOR-CD leads to information loss as compared to NC-CD.

Second, we consider PAM-based channel-coded PNC over block Rayleigh fading channels. In strict delay constrained

wireless communications, a codeword with limited length spans a finite number of  $\mathcal{B}$  fading blocks. Our simulated systems set  $\mathcal{B}$  to be the typical value of 4. With Gray-mapped 4PAM modulations, Fig. 6 illustrates the frame error rate (FER) of XOR-CD, NC-CD and CD-NC with the nonbinary channel coding over  $\mathbb{Z}_4$  at code rates of 1/2 and 1/4, respectively. In this case, NC-CD attempts to decode NC codeword with all possible coefficients which belong to non-zero divisors, i.e.,  $\{(1, 3), (1, 3), (3, 1), (3, 3)\}$ . It can be observed that CD-NC outperforms NC-CD by more than 2 dB at the FER of  $10^{-3}$  for both the code rates. This is because that NC-CD loses the useful information in directly decoding NC message. Also, it is interesting to note that XOR-CD suffers a slight performance loss of 1 dB as compared to NC-CD in this scenario.

#### V. CONCLUSIONS AND FUTURE RESEARCH TRENDS

##### A. Conclusions

In this paper, we conducted a concise overview on the recently proposed channel-coded PNC over TWRC communications and their variant decoders. To be specific, Sect. II introduced the model of channel-coded PNC system and NC mapping design. After that, we presented the different PNC decoders for BICM-PNC and CM-PNC in terms of decoding performance and complexity in Section III. In Sect. IV, we showed simulation results to compare the performance of BICM-PNC and CM-PNC.

##### B. Future Research Trends

- 1) **Rate-diverse Channel-coded PNC:** Most related works in the literature focus on the PNC where the same channel code is used for two users. A recent study investigated different modulations and the same coding rate for two users to realize rate-diverse PNC [13]. Nevertheless, little research attention has been paid to the rate-diverse PNC where two users can employ the channel codes of the different coding rates. This study is expected to further enhance achievable PNC rate especially in TWRCs where two user-to-relay channels have different channel conditions.
- 2) **Multi-user Channel-coded PNC:** Effectively supporting massive connectivity is important to ensure that the forthcoming 5G network can support the Internet of Things (IoT) functionalities. Thus, it is significant to investigate  $K$ -user PNC communication scenarios,  $K \geq 3$ . How to characterize the decoding behavior for  $K$ -user PNC based on the constellation minimum distance [6] is still a challenging task.
- 3) **Design of Channel Codes for PNC:** To approach the PNC channel capacity at high SNR regime, prior works have designed irregular repeat-accumulate codes and irregular LDPC codes for XOR-CD and NC-CD, respectively. Nevertheless, how to explore variants of the channel codes, e.g., convolutional LDPC codes and spatially-coupled LDPC codes, for CD-XOR and CD-NC is certainly to be expected and deserves further work, especially in low-to-medium SNR regime.

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